1. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

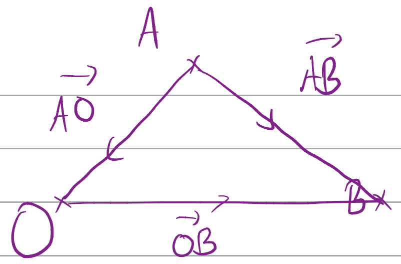
(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

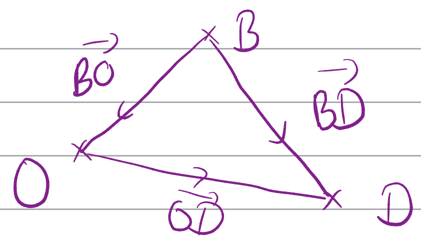
a) $\overrightarrow{AB} = \overrightarrow{BD}$



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix}-2\\-3\\4\end{pmatrix} + \begin{pmatrix}4\\-2\\3\end{pmatrix} = \begin{pmatrix}2\\-5\\7\end{pmatrix} = \overrightarrow{BD}$

$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$



$\overrightarrow{OD} = \overrightarrow{BD} - \overrightarrow{BO}$

$= \overrightarrow{AB} - \overrightarrow{BO}$

$= \begin{pmatrix}2\\-5\\7\end{pmatrix} - \begin{pmatrix}-4\\2\\-3\end{pmatrix} = \begin{pmatrix}6\\-7\\10\end{pmatrix}$

Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

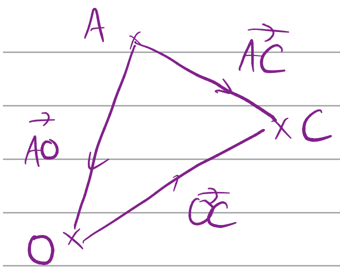
the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)



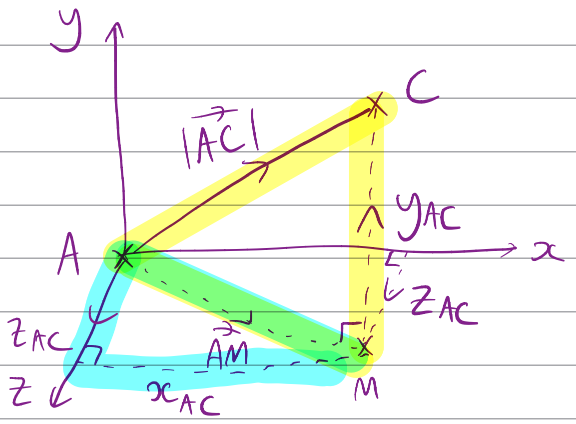
Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

b) $\vec{AC} = \vec{AO} + \vec{OC}$

$$= \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} a \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} a-2 \\ 2 \\ 2 \end{pmatrix}$$



$$a^2 + b^2 = c^2$$

$$|\vec{AM}|^2 + y_{AC}^2 = |\vec{AC}|^2$$

$$|\vec{AM}|^2 = x_{AC}^2 + z_{AC}^2$$

$$x_{AC}^2 + z_{AC}^2 + y_{AC}^2 = |\vec{AC}|^2$$

$$(a-2)^2 + (2)^2 + (2)^2 = |\vec{AC}|^2$$

$$(a-2)^2 + 4 + 4 = 16$$

$$(a-2)(a-2) + 8 - 16 = 0$$

$$a^2 - 2a - 2a + 4 - 8 = 0$$

$$a^2 - 4a - 4 = 0$$

$$a^2 - 4a = 4$$

$$(a-2)^2 - 2^2 = 4$$

$$(a-2)^2 = 8$$

$$a - 2 = \pm\sqrt{8}$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

$$a = 2 \pm \sqrt{8}$$

$$a = 2 - \sqrt{8}$$

$$\Rightarrow \quad \underline{a = 2 - 2\sqrt{2}}$$

2. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

a) $\overrightarrow{AB} = B - A$

$A = \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$

$\Rightarrow \overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$ 1

$\Rightarrow \overrightarrow{AB} = \underline{\underline{\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}}}$ 1

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

b) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix}$

$\Rightarrow \overrightarrow{OC} = 2\overrightarrow{AB}$ 1

$\Rightarrow \overrightarrow{OC}$ is parallel to $\overrightarrow{AB} \Rightarrow OABC$ is a trapezium. 1

3.

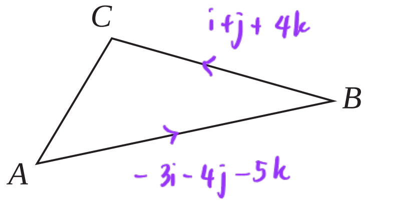


Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

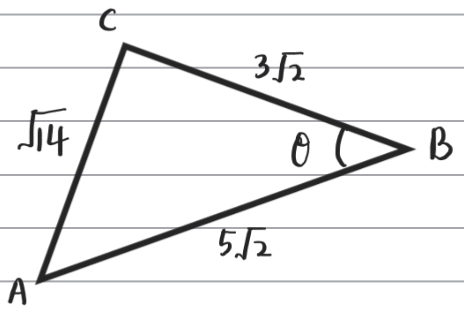
(3)

(a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= -3i - 4j - 5k + i + j + 4k$ (1)

$= -2i - 3j - k$ (1)

(b)



$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2}$

$= 5\sqrt{2}$

$|\overrightarrow{BC}| = \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= 3\sqrt{2}$

$|\overrightarrow{AC}| = \sqrt{(-2)^2 + (-3)^2 + (-1)^2}$

$= \sqrt{14}$

Question continued

$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos ABC$

(1)

$14 = (5\sqrt{2})^2 + (3\sqrt{2})^2 - 2(5\sqrt{2}) \times 3\sqrt{2} \times \cos ABC$ (1)

$= 50 + 18 - 60 \cos ABC$

$60 \cos ABC = 50 + 18 - 14$

$= 54$

$\cos ABC = \frac{54}{60}$

$\cos ABC = \frac{9}{10}$ # (1)

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4.

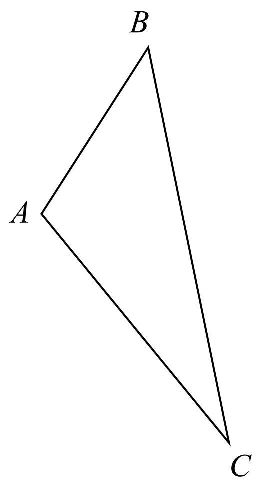

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -9 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 4 \end{pmatrix}$ (1)

$\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$ let $\underline{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 3 \\ -6 \\ 4 \end{pmatrix}$

$|\underline{a}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$, $|\underline{b}| = \sqrt{3^2 + (-6)^2 + 4^2} = \sqrt{61}$ (1)

$\underline{a} \cdot \underline{b} = (2 \times 3) + (3 \times -6) + (1 \times 4) = 6 - 18 + 4 = -8$ (1)

$\Rightarrow \cos\theta = \dfrac{-8}{\sqrt{14} \times \sqrt{61}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-8}{\sqrt{14} \times \sqrt{61}}\right)$ (1)

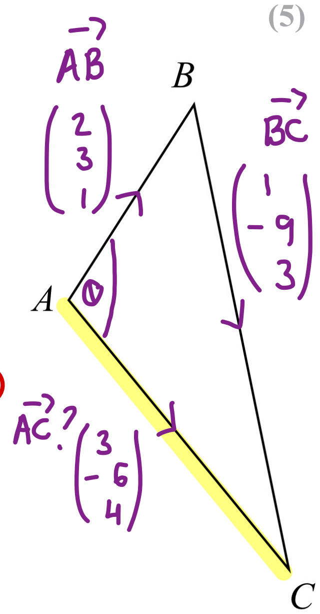

Figure 2

$\Rightarrow \theta = 105.887...$

$\theta = 105.9°$

$\Rightarrow \angle BAC = 105.9°$ as required.